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Analysis of the characteristics of the harmonics coefficient J_2 of the Earth's gravity field in different periods

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The characteristics of the geopotential coefficient J_2 in different periods are analyzed using satellite laser ranging data spanning the last 27 years. The satellites used in the analysis are Lageos1 and Lageos2. The variations in J_2 are obtained by determining the dynamic orbit. The results show that there are strong seasonal and long-term variations. For different data spans, the seasonal variations agree well in terms of both amplitude and phase. Using all the data, the amplitude and phase of the annual term are 2.5×10^{-10} and 127°, respectively, while the amplitude and phase of the semiannual term are 0.94×10^{-10} and 213°, respectively. In the case of long-term variation, the secular variation in J_2 ($\dot{J}_2$) is $-2.2 \times 10^{-11}\ \mathrm{a}^{-1}$ from 1984 to 2010. $\dot{J}_2$ differs for the different periods because of interannual variations, such as the "1998 anomaly". Another anomaly may have taken place during 2007–2010. Although the cause of the anomaly is unknown, it is an important observational constraint on the shape of the Earth.

satellite laser ranging, geopotential, Lageos 1 satellite, Lageos 2 satellite

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The Earth is a complex dynamic system that has a mobile atmosphere and oceans, a varying global distribution of ice, snow and water, and a fluid core that is undergoing some type of hydromagnetic motion. These processes redistribute the Earth's mass and consequently affect the Earth's rotation and its gravitational field over time and space. These variations can be embodied by harmonics coefficients.

A major step forward in geophysics has been progress in investigating the Earth's gravity since the launch of the first artificial satellites. A satellite orbit is sensitive to the Earth's gravity field, and thus, the satellites Starlette, Lageos1 and Lageos2 have been widely used in studying the Earth's geopotential; for example, there have been investigations of the global gravity field model, tidal variations and variations in the low-degree zonal coefficients. The results obtained have played an important role in the field of precision space measurement [1–5]. GRACE, launched in March 2002, is making detailed measurements of the Earth's gravity field, which will lead to discoveries about gravity and the Earth's natural systems. These discoveries could have far-reaching benefits to society and the world's population. However, J_2 data derived from GRACE observations have poor accuracy.

Global satellite laser ranging (SLR) stations are distributed around the world. Lageos1 and Lageos2 are passive SLR targets that have stable orbits, a long observational history and observational accuracy better than 1 cm. These advantages mean that the two satellites are widely used to study the variations in low-degree zonal coefficients. Guo et al. [6], Yoder et al. [7], Nerem et al. [8], Gegout et al. [9], Dong et al. [10], Cheng et al. [11–14], and Cox et al. [15] used a single or multiple SLR satellites to analyze various characteristics of the low-degree zonal coefficients. Because of the complexity of the variations in J_2, this paper investi-

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gates the seasonal and long-term variations in J_2 for different periods and recent variations in J_2.

1 Method

In this study, computations are made for an arc length of 15 days using the SHORDE program developed by Shanghai Astronomical Observatory. The precise satellite ephemeris and the partial derivatives of estimated parameters with respect to satellite position are obtained by first determining the dynamic orbit. The variations in J_2 are then obtained through least-squares adjustment. The estimated parameters are the state vector (position and velocity), the low-degree zonal coefficients, the radiation pressure coefficient per 15 days and the estimated along-track acceleration coefficient per 3 days. The constants, reference system and force models are presented in Table 1.

We let $J_n = -\sqrt{2n+1}\bar{C}_{n0}$ (where $\bar{C}_{n0}$ is normalized, n is the degree), and $\Delta J_n = J_n - J_{n0}$, where J_{n0} is the coefficient of the gravity force model. A satellite's node (Ω) is primarily sensitive to variations in the even zonal harmonic coefficients. The following dynamical equation [16] describes the perturbation in Ω due to the variations in J_2 and J_4:

$$\frac{\mathrm{d}\delta\Omega}{\mathrm{d}t} = -\frac{3}{2}n\left(\frac{R_e}{a}\right)^2 \frac{\cos i}{\left(1-e^2\right)^2} \times \left(\delta J_2 + \delta J_4 f_4 + \cdots\right),$$
$$f_4 = \frac{5}{8}\left(\frac{R_e}{a}\right)^2 \times \left(7\sin^2 i - 4\right) \times \left(\frac{1+\frac{3e^2}{2}}{\left(1-e^2\right)^4}\right), \tag{1}$$

here, $n = 2\pi / P$ is the orbital mean motion, P is the orbit period and R_e is the Earth's radius. For Lageos satellites, the inclination factor $f_4 = 0.37$ is large enough that the magnitude of δJ_4 relative to δJ_2 must be considered. Therefore, in this paper, the estimated low-degree zonal coefficients are J_2, J_3 and J_4.

2 Data

For a single satellite, the obtained low-degree zonal coefficients contain the lumped effects of the high-degree zonalcoefficients of the Earth's gravity field (see eq. (1)). Thus, a method that uses multiple satellites at various altitudes and inclinations is particularly important for separating the low-degree spherical harmonics. SLR data for the periods from January 1984 to December 2010 for Lageos1 and from October 1992 to December 2010 for Lageos2 are used in our study. The main parameters of the two satellites are listed in Table 2.

Table 1 Constants and force models

The force and the parameters	Description
The reference system	ITRF2000
Earth rotation parameter	IERS Bulletin C04
Atmospheric refraction	Marini-Murray
Precession and nutation	IERS2003 convention
N-body perturbation	DE403/LE403
Gravity field model	GGM001C
Ocean tide perturbation	CSR3.0
Solid tide perturbation	IERS2003 convention
Center-of-mass offset	0.251m

Table 2 Main parameters of Lageos1 and Lageos2

Satellite	Laoges1	Lageos2
COSPAR ID	7603901	9207002
Launch data	May 4, 1976	October 22, 1992
RRA Diameter (cm)	60	60
Shape	Sphere	Sphere
Reflectors	426	426
Orbit	Circular	Circular
Inclination (°)	109.84	52.64
Eccentricity	0.0045	0.0135
Perigee (km)	5860	5620
Period (min)	225	223
Weight (kg)	406.965	405.38

3 Seasonal variation

Figures 1 and 2 show the estimated variations in J_2 and the results of frequency analysis, respectively. The two figures illustrate that there are strong annual and semiannual signals with amplitudes of 1.5×10^{-10} and 1.2×10^{-10}, respectively. The seasonal variations are related to the mass redistributions of the atmosphere, ocean and underground water. The atmosphere and water are the dominant contributors at the annual frequency, while the ocean is the main contributor at the semiannual frequency [8,10,13,17,18]. The following equation is used to fit the amplitude and phase of the annual and semiannual terms:

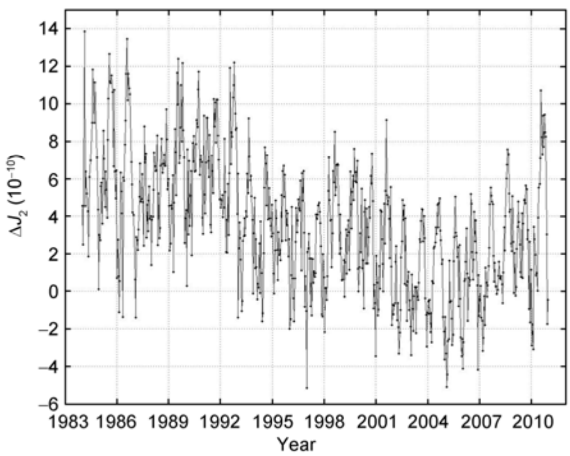

Figure 1 Variations in 15-day J_2 obtained from Lageos1 and Lageos2 SLR data.

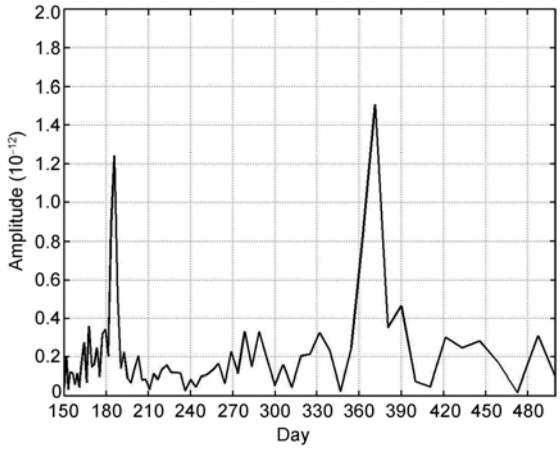

Figure 2 Frequency analysis of ΔJ_2.

$$\Delta J_2 = J_2(t_0) + \dot{J}_2(t-t_0) + A_{sa}\cos(\omega(t-t_0)+\phi_{sa}) + A_{ssa}\cos(2\omega(t-t_0)+\phi_{ssa}), \quad (2)$$

here, $\dot{J}_2$ is the secular variation in J_2, A_{sa} and ϕ_{sa} are the amplitude and phase of the annual term, respectively, A_{ssa} and ϕ_{ssa} are the amplitude and phase of the semiannual term, respectively, and t_0 is January 1, 1984.

The amplitude and phase of J_2 variation were fitted by Gegout et al. [19], Dong et al. [10] and Cheng et al. [13] using SLR data for the periods from 1985 to 1989, from 1984 to 1992 and from 1993 to 1996, respectively. In our study, SLR data for the same three periods are used and all the results are listed in Table 3. The table shows that our results agree well with the results of the other studies at annual and semiannual frequencies, in terms of both amplitude and phase. However, there are still differences due to many factors, such as the different models, data processing methods and definitions of t_0 used. The difference in the definition of t_0 could introduce phase ambiguity of 10° to 15° [13].

SLR data covering the five periods of 1985–1989, 1984–1992, 1993–1996, 1997–2001, 2002–2010 are used separately to fit the amplitude and phase of the annual and semiannual terms. The results for the different periods differ from each other, but the differences are not large. The maximum and minimum differences in the amplitude of the annual variations are 0.51×10^{-10} and 0.02×10^{-10}, respectively, and the maximum and minimum differences in the phase of the annual variations are 21° and 2°, respectively. In the case of semiannual variations, the maximum and minimum differences are respectively 0.54×10^{-10} and 0.04×10^{-10} for the amplitude and 46° and 5° for the phase. Using all the SLR data, the amplitude and phase of the annual term are 2.50×10^{-10} and 127°, respectively, while the amplitude and phase of the semiannual term are 0.94×10^{-10} and 213°, respectively. These values indicate that the character of the seasonal variation in J_2 is related to the data period considered. The comparison of the results for semiannual frequency in various time spans shows less agreement than the results for annual frequency. This is due to uncertainty in the measurement, unmodeled forces perturbing the orbit and semiannual errors in the ocean tide model.

4 Long-term variation

Figure 1 also shows long-term variation in addition to seasonal variation. Yoder et al. [7] found that the secular variation in J_2 ($\dot{J}_2$) was $-3.0 \times 10^{-11}\ \mathrm{a}^{-1}$ using Lageos 1 data spanning an interval of 5 years, and thought that this might be due to postglacial rebound. Cheng et al. [14], Cox et al. [15] and Eanes et al. [20] also analyzed the long-term variation using SLR data obtained from multiple satellites and they agreed that J_2 was decreasing.

Nerem et al. [21], Cazenav et al. [22], Chapanov et al. [23] and Cox et al. [15] used SLR data for the periods of 1986–1994, 1984–1994, 1984–2000 and 1997–2002, separately to fit $\dot{J}_2$. In this paper, SLR data covering the same four periods are used to fit $\dot{J}_2$ according to eq. (2). The

Table 3 Annual and semiannual variations in J_2 derived from SLR data

Source	Time	Solution	Annual variation		Semi-annual variation	
			Amplitude (10^{-10})	Phase (°)	Amplitude (10^{-10})	Phase (°)
Ref. [13]	1993–1996	5 satellites	2.95	130	0.85	231
Ref. [8]	1980–1989	Lageos1	2.68	115	2.53	198
Ref. [19]	1985–1989	Lageos1	3.20	107	1.70	201
Ref. [10]	1984–1992	Lageos1	2.46	119	2.06	205
This study	1985–1989	Lageos1	2.68	140	0.60	212
This study	1984–1992	Lageos1	2.44	123	1.00	204
This study	1993–1996	Lageos1, Lageos2	2.17	121	0.69	239
This study	1997–2001	Lageos1, Lageos2	2.19	136	1.14	198
This study	2002–2010	Lageos1, Lageos2	2.64	142	1.10	244
This study	1984–2010	Lageos1, Lageos2	2.50	127	0.94	213

results are summarized in Table 4. All the studies, including ours, have general agreement for the same period. However, there are differences likely arising from the number of satellites, force model, data processing method and reference system used. Eanes et al. [20] determined that the difference in $\dot{J}_2$ due to the use of different models and constants was $\pm 0.5\times 10^{-12}$.

Table 4 shows that $\dot{J}_2$ is -2.2×10^{-11} a^{-1} when using all the data from 1984 to 2010 and that it differs for various time spans owing to interannual anomaly variations. The trend of change also differs for various time spans; for example, $\dot{J}_2$ is positive in the periods of 1997–2002 and 2007–2010, which means that $\dot{J}_2$ was increasing. Besides these two time periods, the trends were decreasing in other time spans and the differences in $\dot{J}_2$ were about 1.1×10^{-12} a^{-1}.

In this paper, a fit from 1997 to 2002 yields a rate of 2.7×10^{-11} a^{-1}, whereas Cox et al. [15] yielded a rate of 2.2×10^{-11} a^{-1} for the same time span. The two results differ but the trend is the same. $\dot{J}_2$ is -3.3×10^{-11} and -2.5×10^{-11} a^{-1} for the period from 1984 to 1997 and for the period from 1984 to 2002, respectively. These data indicate that there is a large anomaly from 1997 to 2002 named the “1998 anomaly”. Figure 3 shows a positive jump since late 1997, and J_2 reaches a maximum in 2000. The 1998 anomaly was first detected by Cox et al. [15] and subsequently investigated by many researchers [24–26]. Dickey et al. [24] deduced that it was caused primarily by a recent surge in sub-polar glacial melting and the Pacific Ocean. Cheng et al. [14] deduced that the anomaly was due to the superposition of the 5.8-year variation with a decadal variation. Chao et al. [26] found that an oceanographic event that took place in the extratropic north and south Pacific basins matched remarkably well with the 1998 anomaly.

Figure 3 also shows a peak in early 2010. $\dot{J}_2$ is -2.9×10^{-11}, -2.2×10^{-11} and 3.5×10^{-11} a^{-1} in the periods 1984–2007, 1984–2010 and 2007–2010, respectively. This means there might have been another large anomaly from 2007 to 2010. However, the cause of the anomaly is still unknown and remains to be investigated.

5 Conclusions

This paper analyzed seasonal and long-term variations in J_2 for various periods using the latest SLR data provided by Lageos1 and Lageos2 satellites, which are available from January 1984 to December 2010 and from October 1992 to December 2010, respectively. The following conclusions are drawn.

(1) The amplitude and phase of the annual and semiannual variations in J_2 are related to the time span. The

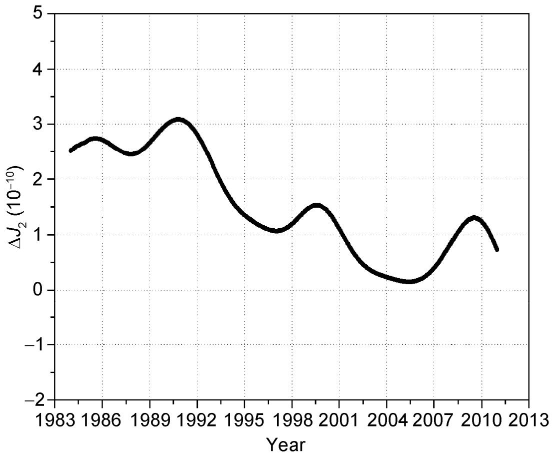

Figure 3 Variations in J_2 after application of a two-year filter.

Table 4 Long-term variations in J_2 obtained from SLR data

Source	Solution	Time	$\dot{J}_2$ (10^{-11})
Ref. [7]	Lageos1	1976–1983	–3.0
Ref. [27]	8 satellites	1976–2008	–2.3
Ref. [15]	10 satellites	1979–1996	–2.8
Ref. [20]	Lageos1	1976–1995	–3.0
Ref. [21]	Lageos1, Lageos2, Ajisai, Starlette	1986–1994	–2.8
Ref. [22]	Lageos1, Lageos2	1984–1994	–3.0
Ref. [23]	Lageos1, Lageos2	1984–2000	–2.7
Ref. [15]	10 satellites	1997–2002	2.2
This study	Lageos1, Lageos2	1984–1994	–2.5
This study	Lageos1, Lageos2	1986–1994	–2.6
This study	Lageos1, Lageos2	1984–2000	–2.9
This study	Lageos1, Lageos2	1997–2002	2.7
This study	Lageos1, Lageos2	1984–1997	–3.3
This study	Lageos1, Lageos2	1984–2002	–2.5
This study	Lageos1, Lageos2	1984–2010	–2.2
This study	Lageos1, Lageos2	1984–2007	–2.9
This study	Lageos1, Lageos2	2007–2010	3.5

results differ for different time spans, but the differences are not large. Using all the SLR data, the amplitude and phase of the annual term are 2.50×10^{-10} and 127°, respectively, while the amplitude and phase of the semiannual term are 0.94×10^{-10} and 213°, respectively. A comparison of our results and the results obtained in other studies showed a general agreement at the annual and semiannual frequencies in terms of both amplitude and phase.

(2) In the case of the long-term variation, the secular variation in J_2 ($\dot{J}_2$) was -2.2×10^{-10} a^{-1} from 1984 to 2010. $\dot{J}_2$ differed for the different time periods because of interannual variations. According to our study, there might have been another large anomaly from 2007 to 2010, which would have been similar to the 1998 anomaly. However, its origin is unexplained and needs to be investigated thoroughly.

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